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Parametrization of the Transverse Momentum Dependent Light-Front Correlator for Gluons

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Abstract We study the transverse momentum dependent light-front correlator for gluons. At the operator level this is expressed as a matrix element containing nonlocal field strength operators and gauge links bridging the nonlocality. We parametrize the leading (twist-2) gluon–gluon correlator in terms of transverse momentum dependent distribution functions for unpolarized, vector and tensor polarized targets (the latter being relevant for spin-1 targets). For a tensor polarized target there are eleven functions among which two are time reversal odd. We discuss bounds on some functions which might become useful for future applications.

1 Introduction

The cross section of high energy processes can be written as a convolution of hadronic matrix elements and partonic hard cross section, so long as the validity of factorization theorems is assumed. Nonlocal forward matrix elements containing quarks and gluon fields are studied in terms of Parton distribution functions (PDFs). Extending the non locality to the transverse directions offers possibilities to study transverse momentum depended (TMD) PDFs, or simply TMDs. The hadronic matrix element containing two gluon fields was parametrized in terms of TMD PDFs for the first time in [1] and later on proposed in a different form in [2]. These works describe unpolarized and vector polarized targets, relevant for spin-0 and spin- $\frac{1}{2}$ particles. Recently in [3] the parametrization of the gluon–gluon correlator has been extended to tensor polarization, allowing in fact the inclusion of spin-1 hadrons in the description. The outline of this work is as follows: in Sect. 2 we show the parametrization of the gluon–gluon correlator in terms of gluon TMDs for hadrons with spin up to 1 and in Sect. 3 we present some bounds on the distributions.

2 Parametrization of the TMD Gluon–Gluon Correlation Function for Hadrons of Spin ≤ 1

The correlation function for gluons in hadrons is defined as:

$$\Gamma^{[U,U']\mu\nu;\rho\sigma}(k, P, n, \dots) \equiv \int \frac{d^4\xi}{(2\pi)^4} e^{ik\cdot\xi} \langle P, \dots | F^{\mu\nu}(0) U_{[0,\xi]} F^{\rho\sigma}(\xi) U'_{[\xi,0]} | P, \dots \rangle, \quad (1)$$

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where a trace in color space (Tr_c) is implicitly assumed. The Wilson lines $U_{[0,\xi]}$ and $U'_{[\xi,0]}$ guarantee color gauge invariance as they bridge the nonlocality of the two gluon field tensors. The dots indicate the spin vectors and/or tensors needed to deal with polarization of hadrons with spin ≤ 1 .

The relevant momentum variables are P and k , i.e. the hadron and parton momentum respectively. We use the lightlike vector n , i.e. $n^2 = 0$, and we parametrize P and k such that they satisfy:

$$P^2 = M^2, \quad P \cdot n = 1, \quad k^\mu = x P^\mu + k_T^\mu + (k \cdot P - x M^2) n^\mu. \quad (2)$$

where M is the mass of the hadron.

The target spin states are characterized by the density matrix. For hadrons of spin 0 and $\frac{1}{2}$, one needs to introduce a spacelike vector S^μ which parametrizes the density matrix [4,5]. By demanding $S^2 = -1$ and $P \cdot S = 0$ one can parametrize S in terms of the dimensionful vectors P and n :

$$S^\mu = S_L \frac{P^\mu}{M} + S_T^\mu - M S_L n^\mu. \quad (3)$$

To extend the description to spin-1 particles a polarization tensor $T^{\mu\nu}$ is needed, which satisfies $P_\mu T^{\mu\nu} = 0$. The parametrization of the tensorial part reads:

$$T^{\mu\nu} = \frac{1}{2} \left[\frac{2}{3} S_{LL} g_T^{\mu\nu} + \frac{4}{3} S_{LL} \frac{P^\mu P^\nu}{M^2} + \frac{S_{LT}^{\{\mu} P^{\nu\}}}{M} + S_{TT}^{\mu\nu} - \frac{4}{3} S_{LL} P^{\{\mu} n^{\nu\}} - M S_{LT}^{\{\mu} n^{\nu\}} + \frac{4}{3} M^2 S_{LL} n^\mu n^\nu \right]. \quad (4)$$

We have used the metric tensor in transverse space defined as $g_T^{\mu\nu} \equiv g^{\mu\nu} - P^{\{\mu} n^{\nu\}}$ (curly brackets denote symmetrization of the indices), with nonvanishing elements $g_T^{11} = g_T^{22} = -1$. The spin tensor is symmetric and traceless and it has five independent parameters, namely S_{LL} , the two components of the transverse vector S_{LT} , and the two independent components of the symmetric traceless transverse tensor S_{TT} .

Looking at Eq. (1) we note that, even though for unspecified processes the path could run along arbitrary paths, we have included a dependence on the lightlike four-vector n , that enters upon consideration of staple-like gauge links running along the light-front ($\xi \cdot n = 0$) via lightlike $\xi \cdot P = \pm\infty$. Integrating Eq. (1) over $k \cdot P$, one obtains the TMD (light-front) correlator

$$\Gamma^{[U,U']\mu\nu;\rho\sigma}(x, \mathbf{k}_T; P, n) \equiv \int \frac{d\xi \cdot P d^2 \xi_T}{(2\pi)^3} e^{ik \cdot \xi} \langle P | F^{\mu\nu}(0) U_{[0,\xi]} F^{\rho\sigma}(\xi) U'_{[\xi,0]} | P \rangle \Big|_{\xi \cdot n = 0}. \quad (5)$$

Simple power counting leads to the definition of the leading (usually referred to as leading twist or twist-2) correlator:

$$\Gamma^{ij}(x, \mathbf{k}_T) \equiv \Gamma^{[U,U']ni;nj}(x, \mathbf{k}_T; P, n). \quad (6)$$

The latter is indeed the operator considered as a starting point in [3], and we repeat the main results on the parametrization therein derived.

In the decomposition we employ symmetric and traceless tensors $k_T^{i_1 \dots i_n}$ that are built from the partonic momentum k_T . Up to rank $n = 4$, these are given by

$$k_T^{ij} \equiv k_T^i k_T^j + \frac{1}{2} \mathbf{k}_T^2 g_T^{ij}, \quad (7)$$

$$k_T^{ijk} \equiv k_T^i k_T^j k_T^k + \frac{1}{4} \mathbf{k}_T^2 \left(g_T^{ij} k_T^k + g_T^{ik} k_T^j + g_T^{jk} k_T^i \right), \quad (8)$$

$$k_T^{ijkl} \equiv k_T^i k_T^j k_T^k k_T^l + \frac{1}{6} \mathbf{k}_T^2 \left(g_T^{ij} k_T^{kl} + g_T^{ik} k_T^{jl} + g_T^{il} k_T^{jk} + g_T^{jk} k_T^{il} + g_T^{jl} k_T^{ik} + g_T^{kl} k_T^{ij} \right) - \frac{1}{8} \mathbf{k}_T^4 \left(g_T^{ij} g_T^{kl} + g_T^{ik} g_T^{jl} + g_T^{il} g_T^{jk} \right), \quad (9)$$

satisfying

$$g_{Tij} k_T^{ij} = g_{Tij} k_T^{ijk} = g_{Tij} k_T^{ijkl} = 0. \quad (10)$$

From this decomposition of the Lorentz structures it follows that the functions involved in the parametrization are twist-2 TMDs *of definite rank*. This has the advantage that there is a one-to-one correspondence between the functions in the transverse coordinate \mathbf{b}_T space and the \mathbf{k}_T space, which turns out to be a very important feature when the TMD evolution equations have to be considered [6].

Separation into different polarizations gives:

$$\Gamma = \Gamma_U + \Gamma_L + \Gamma_T + \Gamma_{LL} + \Gamma_{LT} + \Gamma_{TT}, \quad (11)$$

where:

$$\Gamma_U^{ij}(x, \mathbf{k}_T) = \frac{x}{2} \left[-g_T^{ij} f_1(x, \mathbf{k}_T^2) + \frac{k_T^{ij}}{M^2} h_1^\perp(x, \mathbf{k}_T^2) \right], \quad (12)$$

$$\Gamma_L^{ij}(x, \mathbf{k}_T) = \frac{x}{2} \left[i\epsilon_T^{ij} S_L g_1(x, \mathbf{k}_T^2) + \frac{\epsilon_T^{[i} k_T^{j]\alpha} S_L}{2M^2} h_{1L}^\perp(x, \mathbf{k}_T^2) \right], \quad (13)$$

$$\Gamma_T^{ij}(x, \mathbf{k}_T) = \frac{x}{2} \left[-\frac{g_T^{ij} \epsilon_T^{S_T k_T}}{M} f_{1T}^\perp(x, \mathbf{k}_T^2) + \frac{i\epsilon_T^{ij} \mathbf{k}_T \cdot \mathbf{S}_T}{M} g_{1T}(x, \mathbf{k}_T^2) \right. \\ \left. - \frac{\epsilon_T^{k_T[i} S_T^{j]} + \epsilon_T^{S_T[i} k_T^{j]}}{4M} h_1(x, \mathbf{k}_T^2) - \frac{\epsilon_T^{[i} \alpha k_T^{j]\alpha} S_T}{2M^3} h_{1T}^\perp(x, \mathbf{k}_T^2) \right], \quad (14)$$

$$\Gamma_{LL}^{ij}(x, \mathbf{k}_T) = \frac{x}{2} \left[-g_T^{ij} S_{LL} f_{1LL}(x, \mathbf{k}_T^2) + \frac{k_T^{ij} S_{LL}}{M^2} h_{1LL}^\perp(x, \mathbf{k}_T^2) \right], \quad (15)$$

$$\Gamma_{LT}^{ij}(x, \mathbf{k}_T) = \frac{x}{2} \left[-\frac{g_T^{ij} \mathbf{k}_T \cdot \mathbf{S}_{LT}}{M} f_{1LT}(x, \mathbf{k}_T^2) + \frac{i\epsilon_T^{ij} \epsilon_T^{S_{LT} k_T}}{M} g_{1LT}(x, \mathbf{k}_T^2) \right. \\ \left. + \frac{S_{LT}^{[i} k_T^{j]}}{M} h_{1LT}(x, \mathbf{k}_T^2) + \frac{k_T^{ij\alpha} S_{LT\alpha}}{M^3} h_{1LT}^\perp(x, \mathbf{k}_T^2) \right], \quad (16)$$

$$\Gamma_{TT}^{ij}(x, \mathbf{k}_T) = \frac{x}{2} \left[-\frac{g_T^{ij} k_T^{\alpha\beta} S_{TT\alpha\beta}}{M^2} f_{1TT}(x, \mathbf{k}_T^2) + \frac{i\epsilon_T^{ij} \epsilon_T^{\beta\gamma} k_T^{\gamma\alpha} S_{TT\alpha\beta}}{M^2} g_{1TT}(x, \mathbf{k}_T^2) \right. \\ \left. + S_{TT}^{ij} h_{1TT}(x, \mathbf{k}_T^2) + \frac{S_{TT\alpha}^{[i} k_T^{j]\alpha}}{M^2} h_{1TT}^\perp(x, \mathbf{k}_T^2) \right. \\ \left. + \frac{k_T^{ij\alpha\beta} S_{TT\alpha\beta}}{M^4} h_{1TT}^{\perp\perp}(x, \mathbf{k}_T^2) \right]. \quad (17)$$

Throughout the paper, the dependence of functions on the gauge link is understood and we define $\mathbf{k}_T \cdot \mathbf{S}_T = -\mathbf{k}_T \cdot \mathbf{S}_T$. The transverse antisymmetric tensor is defined as $\epsilon_T^{\mu\nu} \equiv \epsilon^{Pn\mu\nu}$, with nonzero components $\epsilon_T^{12} = -\epsilon_T^{21} = 1$. The functions h_{1L}^\perp , f_{1T}^\perp , h_1 , and h_{1T}^\perp are odd under naive time reversal transformations (T -odd). The functions h_{1LL}^\perp , f_{1LT} , h_{1LT} , h_{1LT}^\perp , f_{1TT} , h_{1TT}^\perp , $h_{1TT}^{\perp\perp}$ are T -even whereas g_{1LT} and g_{1TT} are T -odd and they have been first introduced in [3]. The function f_{1LL} was also called b_1 in the quark case [7], and h_{1TT} shows up in the structure function $\Delta(x, Q^2)$ discussed in Ref. [8] and is called $\Delta_2 G(x)$ in [9]. They are rank-0 functions which survive in the collinear case, i.e. upon integration over transverse momenta.

3 Positivity Bounds on the Gluon Distributions in Spin-1 Targets

As already done in [10] one can impose positivity constraints to the hadronic tensor and find a probabilistic interpretation for some of the distribution functions. Bounds on the TMDs have been studied in [1] for gluons in spin- $\frac{1}{2}$ hadrons and in [7, 11] for quarks in hadrons with spin up to 1.

The correlator Γ^{ij} in (6) is a 2×2 matrix in the transverse gluon polarizations. On the other hand, the spin states of a spin-1 particle are described by the density matrix, which can be written in terms of the independent

components of the vector S and the tensor T . In fact one needs a Cartesian basis of 3×3 matrices, consisting of the identity matrix I , the three-dimensional (generalized) Pauli matrices Σ^i , and the five bilinear combinations $\Sigma^{ij} \equiv \frac{1}{2}(\Sigma^i \Sigma^j + \Sigma^j \Sigma^i) - \frac{2}{3}I\delta^{ij}$. The density matrix (in the rest frame) takes the form:

$$\rho = \frac{1}{3} \left(I + \frac{3}{2} S^i \Sigma^i + 3 T^{ij} \Sigma^{ij} \right), \quad (18)$$

$$\rho = \begin{pmatrix} \frac{S_L}{2} + \frac{S_{LL}}{3} + \frac{1}{3} & \frac{S_{LTx} - i S_{LTy}}{2\sqrt{2}} + \frac{S_{Tx} - i S_{Ty}}{2\sqrt{2}} & \frac{1}{2}(S_{TTxx} - i S_{TTxy}) \\ \frac{S_{LTx} + i S_{LTy}}{2\sqrt{2}} + \frac{S_{Tx} + i S_{Ty}}{2\sqrt{2}} & \frac{1}{3} - \frac{2S_{LL}}{3} & \frac{-S_{LTx} + i S_{LTy}}{2\sqrt{2}} + \frac{S_{Tx} - i S_{Ty}}{2\sqrt{2}} \\ \frac{1}{2}(S_{TTxx} + i S_{TTxy}) & \frac{-S_{LTx} - i S_{LTy}}{2\sqrt{2}} + \frac{S_{Tx} + i S_{Ty}}{2\sqrt{2}} & -\frac{S_L}{2} + \frac{S_{LL}}{3} + \frac{1}{3} \end{pmatrix}. \quad (19)$$

This allows one to write the correlator in a form which explicitly singles out the hadron spin, namely:

$$\Gamma^{ij}(x, \mathbf{k}_\perp; P, S, T) = \sum_{s, s'} \rho_{s', s}(S, T) \Gamma_{s, s'}^{ij}(x, \mathbf{k}_\perp; P) \quad (20)$$

The matrix $\Gamma_{s, s'}^{ij}$ in the right hand of (20) is a 6×6 matrix in the gluon \otimes hadron spin space:

$$\Gamma_{s, s'}^{ij} = \begin{pmatrix} A & B \\ B^\dagger & C \end{pmatrix} \quad (21)$$

where A , B and C are 3×3 blocks whose complete expressions will be given elsewhere. From symmetry considerations, it follows that the block C is the charge-conjugated of the block A and the off-diagonal block are hermitian-conjugate (see Ref. [3] for more details on the hermiticity, time-reversal and charge-conjugation properties of the gluon correlator). We give here the explicit form of the matrix (21) in the collinear case where the blocks are:

$$A = \begin{pmatrix} f_1 + \frac{f_{1LL}}{2} - g_1 & 0 & 0 \\ 0 & f_1 - f_{1LL} & 0 \\ 0 & 0 & f_1 + \frac{f_{1LL}}{2} + g_1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2h_{1TT} & 0 & 0 \end{pmatrix} \\ C = \begin{pmatrix} f_1 + \frac{f_{1LL}}{2} + g_1 & 0 & 0 \\ 0 & f_1 - f_{1LL} & 0 \\ 0 & 0 & f_1 + \frac{f_{1LL}}{2} - g_1 \end{pmatrix} \quad (22)$$

We note that $\Gamma_{s, s'}^{ij}$ is a positive semi-definite matrix, i.e. for each vector in the combined gluon \otimes target spin space it satisfies $v^\dagger \Gamma v \geq 0$.

$$\Gamma_{s, s'}^{ij}(x; P) = \int \frac{d\xi \cdot P}{2\pi} e^{ik \cdot \xi} \langle P, S, T | F^{ni}(0) F^{nj}(\xi) | P, S, T \rangle \Big|_{\xi \cdot n = \xi_T = 0} \\ = \sum_m \langle P_m | F^{ni}(0) | P, s \rangle^* \langle P_m | F^{nj}(0) | P, s' \rangle \delta(P_m \cdot n - (1-x)P \cdot n) \quad (23)$$

This is true for the unintegrated version of Γ as well (see [1, 5]). The matrix (21) is positive semi-definite if its eigenvalues are all ≥ 0 . Together with the requirement for all the diagonal elements to be positive, we find the following bounds for the integrated gluon distribution functions:

$$f_1 \geq f_{1LL}, \\ |g_1| \leq f_1 + \frac{f_{1LL}}{2} \leq \frac{3}{2} f_1 \\ |h_{1TT}| \leq \frac{1}{2} \left(f_1 + \frac{f_{1LL}}{2} + g_1 \right). \quad (24)$$

Including transverse momenta produces many more relations on the functions which will be shown in a future work.

4 Conclusions

We have presented the parametrization of the gluon–gluon correlator in terms of TMDs and we have considered targets with spin up to 1. Some of the new functions presented in [3] survive under integration over transverse momenta and we derived the bounds fulfilled by these collinear distributions.

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References

1. P.J. Mulders, J. Rodrigues, Transverse momentum dependence in gluon distribution and fragmentation functions. *Phys. Rev. D* **63**, 094021 (2001). [[arXiv:hep-ph/0009343](#)]
2. S. Meissner, A. Metz, K. Goeke, Relations between generalized and transverse momentum dependent parton distributions. *Phys. Rev. D* **76**, 034002 (2007). [[arXiv:hep-ph/0703176](#) [HEP-PH]]
3. D. Boer, S. Cotogno, T. van Daal, P.J. Mulders, A. Signori, Y.J. Zhou, Gluon and Wilson loop TMDs for hadrons of spin ≤ 1 . *JHEP* **1610**, 013 (2016). [[arXiv:1607.01654](#) [hep-ph]]
4. E. Leader, Spin in particle physics. *Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol.* **15**, 1 (2011)
5. A. Bacchetta, P.J. Mulders, Deep inelastic leptonproduction of spin-one hadrons. *Phys. Rev. D* **62**, 114004 (2000). [[arXiv:hep-ph/0007120](#)]
6. A. Signori, Flavor and evolution effects in TMD phenomenology: manifestation of Hadron structure in high-energy scattering processes (2016), <https://inspirehep.net/record/1493030/files/Thesis-2016-Signori.pdf>. Accessed 14 Feb 2017
7. A. Bacchetta, Probing the transverse spin of quarks in deep inelastic scattering. [arXiv:hep-ph/0212025](#)
8. R.L. Jaffe, A. Manohar, Nuclear gluonometry. *Phys. Lett. B* **223**, 218 (1989)
9. X. Artru, M. Mekhfi, Transversely polarized parton densities, their evolution and their measurement. *Z. Phys. C* **45**, 669 (1990)
10. W. Lu, J.J. Yang, Bounding the Levelt–Mulders asymmetry from the positivity of the hadronic tensor and Callan–Gross relation. *Z. Phys. C* **73**, 689 (1997). [[arXiv:hep-ph/9601255](#)]
11. A. Bacchetta, M. Boglione, A. Henneman, P.J. Mulders, Bounds on transverse momentum dependent distribution and fragmentation functions. *Phys. Rev. Lett.* **85**, 712 (2000). [[arXiv:hep-ph/9912490](#)]